a specified convergence tolerance. Third, the results indicate that the dichotomic basis approach may be extendible to problems that evolve on multiple timescales.

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Fuzzy-Logic-Based Closed-Loop Optimal Law for Homing Missiles Guidance

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I. Introduction

THE usefulness of optimal control is sharply divided between two distinct classes of dynamic systems, namely, linear systems and nonlinear systems. For linear systems, the theory is complete in the sense that given a quadratic cost and closed-loop or feedback, a guidance law may be determined.[1] For nonlinear systems, generally the best one can do is to determine an open-loop guidance law. Research efforts have produced many numerical algorithms for an open-loop solution for such problems using digital computers. The main disadvantage of these algorithms is that they generally converge slowly and are not suitable for real-time applications. In an open-loop solution, the control at any time instant is not explicitly determined by the states of the system at that time instant. It is well known that a system with an open-loop controller can be sensitive to noise and external disturbances. In contrast, closed-loop control, in which the control is a function of the instantaneous states of the system, is generally robust with respect to such disturbances. Unfortunately, only rarely is it feasible to determine the feedback law for nonlinear systems of any practical significance.[1]

Over the past four decades, a considerable number of homing missile guidance laws have been proposed. One of the most widely used methods is the proportional navigation guidance (PNG) law.[2] The simplicity of the PNG law has been widely recognized. Furthermore, Ho et al.[3] have shown that the conventional PNG law is optimal in the sense that it drives the miss distance to zero while minimizing the integral of the square of missile acceleration. Most existing missiles are guided by PNG law; however, the linear-quadratic guidance rule contains PNG as a particular case for linear state equations, and most envisioned missile engagements exceed these limits because of high tangential and normal accelerations.

As compared to numerical or linearized methods for solving complex optimization problems, in this study a new approach has been adopted to synthesize nonlinear feedback laws. The motivation comes from the field of fuzzy logic. The basic feature of a fuzzy-logic-based controller is that the control strategy can be simply expressed by a set of fuzzy IF–THEN rules that describe the behavior of controller by employing linguistic terms. From these rules, the proper control action is then inferred. In addition, fuzzy-logic-based controllers are relatively easy to develop and simple to implement.

Fuzzy modeling of control systems and fuzzy optimal control have been studied in the past few years.[4] However, we find only a few papers that explicitly consider closed-loop optimal control in homing missile guidance. In Ref. 5, homing guidance schemes based on fuzzy logic have been developed for a planar engagement model. In Ref. 5, two versions of fuzzy guidance schemes have been proposed, the first one using information required for proportional navigation (PN) and the second one using the information required for augmented PN (APN). Then the performances of the two fuzzy guidance schemes, in terms of commanded acceleration profiles and the value of the terminal miss distance, have been compared with both PN and APN.

The purpose of the present Note is to synthesize an optimal closed-loop guidance law for homing missile against a target in a planar interception. The analysis is based on the exact nonlinear equations of motion. Here, exact open-loop optimal control data (not PN or APN data) are used to generate fuzzy rules. The new method is then used effectively in a real-time feedback guidance method. Numerical example demonstrating trajectories obtained by the optimal, fuzzy logic guidance (FLG) and PNG solutions are presented and followed by conclusions.

II. Problem Statement

The geometry used to define the interception problem is shown in Fig. 1. The XY coordinate system represents an inertial frame, and the X axis is along the line of sight at $t = 0$. The target, located at $X_T = X_0$ for $t = 0$, is moving along a straight line that makes an angle β with respect to the X axis. The constant-speed missile is launched at an angle θ_0 relative to the X axis, and the velocity direction $\theta(t)$ is changed by controlling the normal acceleration $a_n(t)$. The optimal intercept problem is stated as follows.

Find the normal acceleration history a_n that minimizes the performance index

$$J = \frac{1}{2}\int_{t_0}^{t_f} a_n^2\,\mathrm{d}t \qquad (1)$$

If $x = x_T - x_M$ and $y = y_T - Y_M$, then differential constraints become

$$\dot{x} = V_T \cos\beta - V_M \cos\theta \qquad (2)$$

$$\dot{y} = V_T \sin\beta - V_M \sin\theta \qquad (3)$$

$$\dot{\theta} = a_n / V_M \qquad (4)$$

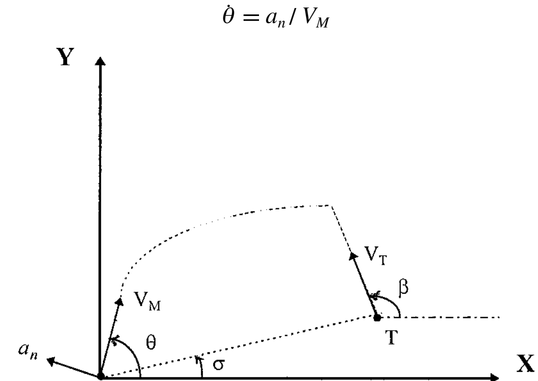

Fig. 1 Missile and target interception geometry.

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with following prescribed boundary conditions:

$$t_0 = 0, \qquad x_T(t_0) = X_0$$
$$y_T(t_0) = x_M(t_0) = y_M(t_0) = 0, \qquad \theta(t_0) = \theta_0 \tag{5}$$

Interception is defined by

$$x_f = 0, \qquad y_f = 0 \tag{6}$$

A particular case for small line-of-sight angles and negligible acceleration along the line of sight (linear state equations) concludes the PNG law and can be stated as[3]

$$a_n = N V_M \dot{\sigma} \tag{7}$$

where a_n is the commanded acceleration (meter per second squared) normal to missile velocity, N is a unitless designer chosen gain (usually in the range of 3–5) known as the navigation constant, V_M is the missile velocity (meter per second), and σ is the line-of-sight angle (radian).

III. Implementation of FLG Law and Simulation Results

A two-dimensional missile–target engagement simulation was set up using differential equation derived in the preceding section. The simulation inputs are initial locations of the missile and target and their velocities. The computation for an open-loop trajectory can be accomplished using MATLAB® optimization toolbox software. We selected three scenarios in which, first, $a_n > 0$ and, second and third, $a_n < 0$; these are (in degree)

$$\theta_0 = 0, \qquad \beta_0 = 120, \qquad \theta_0 = 90, \qquad \beta_0 = 180$$
$$\theta_0 = 90, \qquad \beta_0 = 120$$

In these three scenarios $V_M = 600$ m/s, $V_T = 300$ m/s, and $X_0 = 10{,}000$ m. Note that these sample cases are completely arbitrary, and it is possible to choose other ones. If the number of scenarios and training vectors increase, then better results will be obtained. Then we generated the FLG using data generated with the three scenarios. The method is used by Ref. 6, with five linguistic variables for input $\dot{\sigma}$ as well as decision variable a_n.

Now the FLG can be used to generate a real-time control a_n with the given current state $\dot{\sigma}$ of the missile and target motion. Results of the optimal and PNG trajectories [missile normal accelaration from Eq. (7) with $N = 3$] and that generated by the FLG are compared in Table 1. We can see that results for optimal and FLG solution are close. This is expected because the FLG has been trained with these scenarios. Now, we test the following scenario, which is beyond the region of the state space in which the FLG was trained: $\theta_0 = 80$ deg, $\beta_0 = 110$ deg, $V_M = 500$ m/s, $V_T = 300$ m/s, and $X_0 = 7500$ m.

Figures 2 and 3 show trajectories and controller output a_n of these methods. It is obvious that the FLG is excellent in spite of its relatively simple architecture.

We also determined the floating-point operations (FLOPs) that return the cumulative number of FLOPs for simulation runs using the MATLAB command. Table 1 shows that FLG takes fewer FLOPs than the optimal and PNG do to solve the problem.

Table 1 Comparison of different guidance methods

Scenario inputs	Solution method	t_f, s	Index m^2/s^3	FLOPs
θ_0 = 0 deg, β_0 = 120 deg,	Optimal	14.76	7,641	4.08e+9
X_0 = 10,000 m, V_M = 600 m/s,	PNG	14.84	7,912	77,168
V_T = 300 m/s	FLG	14.76	7,704	75,276
θ_0 = 90 deg, β_0 = 180 deg,	Optimal	13.35	96,175	3.14e+9
X_0 = 10,000 m, V_M = 600 m/s,	PNG	14.84	112,380	77,168
V_T = 300 m/s	FLG	13.35	96,234	68,085
θ_0 = 90 deg, β_0 = 120 deg,	Optimal	16.55	36,240	2.76e+9
X_0 = 10,000 m, V_M = 600 m/s,	PNG	17.58	45,040	91,416
V_T = 300 m/s	FLG	16.48	36,625	84,048
θ_0 = 80 deg, β_0 = 110 deg,	Optimal	15.75	13,458	2.35e+9
X_0 = 7,500 m, V_M = 500 m/s,	PNG	16.31	16,648	84,812
V_T = 300 m/s	FLG	15.48	14,002	78,948

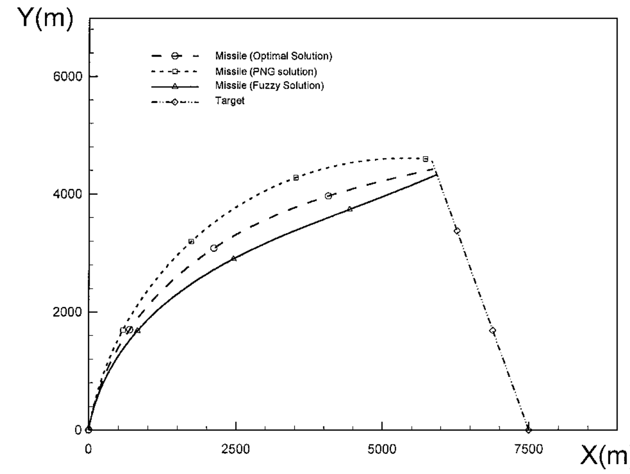

Fig. 2 Trajectories of missile and target; θ_0 = 80 deg, β_0 = 110 deg, V_M = 500 m/s, V_T = 300 m/s, and X_0 = 7500 m.

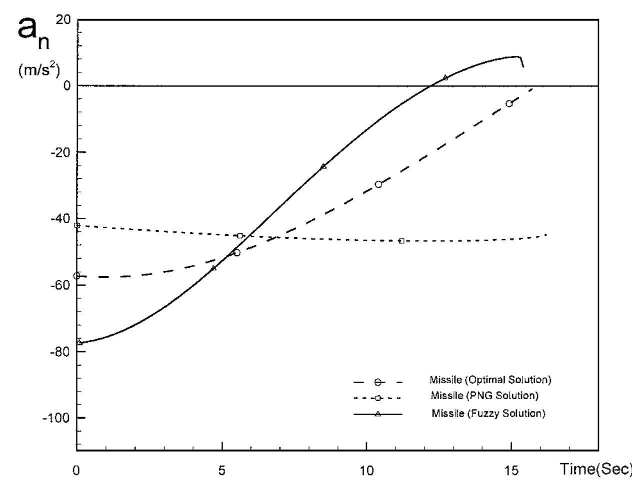

Fig. 3 Missile normal acceleration or control variable; θ_0 = 80 deg, β_0 = 110 deg, V_M = 500 m/s, V_T = 300 m/s, and X_0 = 7500 m.

IV. Conclusion

A new approach is explored to synthesize closed-loop optimal guidance laws for nonlinear dynamic systems using fuzzy logic. Exact open-loop optimal control data from the computed time histories of the state and control variables are used to generate fuzzy rules. The proposed method is then used effectively in a real-time closed-loop law. The efficacy and details of this approach have been successfully demonstrated on an optimal planar interception problem in homing missiles guidance. If the methodology is found to scale up well with many state and control variables to real-life problems, it would have significant potential for a variety of guidance and control problems.

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